25. $f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} \cdots f^{(n)}(0)\frac{x^n}{n!}$

$f(x) = e^{\frac{x}{2}}$ $\quad f(0) = 1$

a) $f'(x) = \frac{1}{2}e^{\frac{x}{2}}$ $\quad f'(0) = \frac{1}{2}$

$f''(x) = \frac{1}{4}e^{x/2}$ $\quad f''(0) = \frac{1}{4}$

$e^{x/2} = 1 + \frac{1}{2}x + \frac{1}{4}\frac{x^2}{2!} \cdots \frac{1}{2^n}\frac{x^n}{n!}$

$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots \frac{x^n}{n!} \cdots$

substitute

$f(x) = e^{x/2} = 1 + \frac{x}{2} + \frac{\left(\frac{x}{2}\right)^2}{2!} + \frac{\left(\frac{x}{2}\right)^3}{3!} \cdots + \frac{\left(\frac{x}{2}\right)^n}{n!}$

b) $g(x) = \frac{e^x - 1}{x} = \frac{x + \frac{x^2}{2} + \frac{x^3}{3!} \cdots \frac{x^n}{n!}}{x}$

$g(x) = 1 + \frac{x}{2} + \frac{x^2}{3!} + \cdots + \frac{x^{n-1}}{n!}$

c) $g'(x) = \frac{1}{2} + \frac{2x}{3!} + \frac{3x^2}{4!} \cdots (n-1)\frac{x^{n-2}}{n!}$

$g'(1) = \frac{1}{2} + \frac{2}{3!} + \frac{3}{4!} \cdots \frac{n-1}{n!} = \sum_{n=1}^{\infty} \frac{n}{(n+1)!}$

$g'(x) = \frac{xe^x - (e^x - 1)\cdot 1}{x^2}$

$g'(1) = \frac{e - e + 1}{1} = 1$

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$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2} + \frac{f'''(0)x^3}{3!} \cdot \frac{f^{(10)}(0)x^{10}}{10!}$

24. $f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} \cdot \frac{x^{10}}{11!}$

a) $f'(0) = \frac{1}{2}$

$f^{(10)}(0) = \frac{10!}{11!}$ $\quad \frac{f^{(10)}(0)\cancel{x^{10}}}{\cancel{10!}} = \frac{\cancel{x^{10}}}{11!}$ $\quad 10!$

$f^{(10)}(0) = \frac{1}{11}$

b) $g(x) = x\,f(x) = x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots \frac{x^n}{n!}$

c) $x f(x) = e^x - 1$

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9.2b Taylor Series

Maclaurin series you should have memorized:

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \cdots \frac{x^n}{n!} \cdots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots \frac{x^{2n+1}}{(2n+1)!}(-1)^n$

$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} \cdots \frac{x^{2n}}{(2n)!}(-1)^n$ n starts at 0

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots \frac{x^n}{n}(-1)^{n+1}$

n starts at 1

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Taylor series for f(x) centered on x=a

$$f(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2} + f'''(a)\frac{(x-a)^3}{3!} + \cdots + f^{(n)}(a)\frac{(x-a)^n}{n!} \cdots$$

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Find the Taylor series generated by f(x)=e^x at x=2

$f(x)=e^x \quad f(2)=e^2$

$f'(x)=e^x \quad f'(2)=e^2$

$f''(x)=e^x \quad f''(2)=e^2$

$\vdots \qquad \vdots$

$$e^x = e^2 + e^2(x-2) + e^2\frac{(x-2)^2}{2} + \frac{e^2(x-2)^3}{3!} \cdots + \frac{e^2(x-2)^n}{n!}$$

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Find the Maclaurin series for $f(x)=\frac{1+\cos(2x)}{2}$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots \frac{x^{2n}}{(2n)!}(-1)^n$$

$$\frac{1+\cos(2x)}{2} = \frac{1+1}{2} - \frac{(2x)^2}{2 \cdot 2!} + \frac{(2x)^4}{2 \cdot 4!} \cdots \frac{(2x)^{2n}}{2\cdot(2n)!}(-1)^n$$

$$= 1 - 2\cdot\frac{x^2}{2!} + \frac{2^3 x^4}{4!} + \cdots \frac{2^{2n-1}x^{2n}}{(2n)!}(-1)^n$$

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Find the first four nonzero terms for the Maclaurin series for $\sin(x^2)$
Use this series to find a corresponding series for $\int \sin(x^2)\,dx$
(why not just evaluate $\int \sin(x^2)\,dx$?)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots \frac{x^{2n+1}}{(2n+1)!}(-1)^n$$

$$\sin(x^2) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} \cdots \frac{(x^2)^{2n+1}}{(2n+1)!}(-1)^n$$

$$\int \sin(x^2) = \int x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} \cdots \frac{x^{4n+2}}{(2n+1)!}(-1)^n$$

$$\int \sin(x^2)\,dx = \frac{x^3}{3} - \frac{x^7}{7\cdot 3!} + \frac{x^{11}}{11\cdot 5!} \cdots \frac{x^{4n+3}(-1)^n}{(4n+3)(2n+1)!}$$

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